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ON OPTIMAL STABILIZATION OF GYROSTAT ROTARY MOTION

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A. V. GORSHKOV
(Sverdlovsk)
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A solution of the problem of optimal stabilization of rotary motion of a gyrostat whose center of mass moves on an elliptic orbit in a central Newtonian force field is derived. A method of successive approximations for the determination of optimal control is established.

The problem of gyrostat stabilization with its center of mass moving on a circular orbit was solved in [1].

1. Let us consider an axisymmetric gyrostat with three flywheels moving in a central Newtonian force field (O_1 and o are, respectively, the centers of attraction and of the gyrostat center of mass, Fig. 1). We shall investigate the relative motion of the gyrostat without taking into account its effect on the motion of the center of mass which is assumed specified (a bounded problem).

We use the following coordinate systems: system $ox_1x_2x_3$ rigidly attached to the gyrostat whose axes coincide with the principal central axes of inertia, with flywheel axes lying on these axes; the inertial system $O_1X_1X_2X_3$ with $X_1O_1X_2$ the orbit plane; the Koenig axes $ox_1'x_2'x_3'$ whose ox_3' axis parallel to ox_3 is the axis of symmetry, and axes ox_1' and ox_2' in the plane x_1ox_2 . In steady motion axes ox_1' and ox_2' are parallel to axes O_1X_1 and O_1X_2. The spherical system of coordinates R, Φ, Ψ, in which R is the distance between points O_1 and o, Ψ is the angle between vector O_1o and the plane $X_1O_1X_2$, Φ is the angle between the O_1X_1-axis and the projection of vector O_1o on the plane $X_1O_1X_2$, is related to the inertial coordinate system by formulas

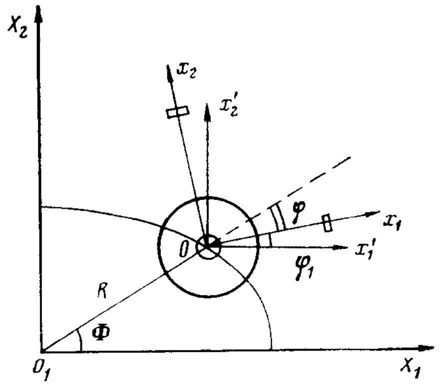

Fig. 1

$$X_1 = R\cos\Phi\cos\Psi,\quad X_2 = R\sin\Phi\cos\Psi,\quad X_3 = R\sin\Psi$$

These coordinate systems are shown in Fig. 1, where φ_1 is the angle between axes ox_1 and ox_1', and φ the angle between the plane X_3O_1o and the ox_1-axis.

The principal central moments of inertia of the gyrostat relative to axes $ox_1x_2x_3$ are denoted by $C_1 = C_2 = C$ and C_3, and the moments of inertia of flywheels by $I_1 = I_2 = I$ and I_3.

We assume that the gyrostat center of mass moves on an elliptic orbit with one of its foci at point O_1. Motion of the gyrostat center of mass is then defined in spherical coordinates by formulas

$$R = \frac{P}{1 + e\cos\Phi}, \quad \Phi^{\cdot} = \frac{\sqrt{\varkappa P}}{P^2}(1 + e\cos\Phi)^2$$
$$\Psi \equiv \Psi^{\cdot} \equiv 0, \quad \varkappa = \mu M_1$$

where P is the orbit parameter, e its eccentricity, μ is the gravitational constant, and M_1 the mass of the attracting center.

The equations of the gyrostat relative motion admit a uniform rotation at the relative velocity ω about the symmetry axis ox_3 normal to the orbit plane; the two flywheels whose axes are in the x_1ox_2 plane are immobilized [1, 2].

Projections of the body instantaneous angular velocity p_1, p_2, p_3 on axes $ox_1x_2x_3$ and of q_1, q_2, q_3 on axes $ox_1'x_2'x_3'$ are connected by the relations

$$p_1 = q_1\cos\varphi_1 + q_2\sin\varphi_1, \quad p_2 = -q_1\sin\varphi_1 + q_2\cos\varphi_1$$
$$p_3 = q_3 + \varphi_1^{\cdot}, \quad \varphi_1^{\cdot} = \varphi^{\cdot} + \Phi^{\cdot}\beta_{33}$$

where β_{ij} are the directional cosines the system of coordinates $ox_1'x_2'x_3'$ relative to $O_1X_1X_2X_3$.

The gravitational forces are determined by the force function whose approximate expression is of the form [1, 2]

$$U = \frac{\varkappa M}{R} + \frac{1}{2}\frac{\varkappa}{R^3}(C_3 - C) - \frac{3}{2}\frac{\varkappa(C_3 - C)}{R^5}\Big(\sum_{i=1}^{3} X_i\beta_{i3}\Big)^2$$

where M is the gyrostat mass.

Equations of the gytostat relative motion in Koenig's axes are of the form [1, 2]

$$\begin{aligned}
&Cq_1^{\cdot} + (C_3 - C)q_2q_3 + C_3\varphi_1^{\cdot}q_2 + q_2g_3 - q_3g_2 + g_1^{\cdot} = M_{x_1'} \qquad (1.1)\\
&Cq_2^{\cdot} + (C - C_3)q_1q_3 - C_3\varphi_1^{\cdot}q_1 + g_1q_3 - g_3q_1 + g_2^{\cdot} = M_{x_2'}\\
&C_3(q_3 + \varphi_1^{\cdot})^{\cdot} + q_1g_2 - q_2g_1 + g_3^{\cdot} = M_{x_3'}\\
&g_1^{\cdot} + Iq_1^{\cdot} + (g_2 + Iq_2)\varphi_1^{\cdot} = w_1,\\
&g_2^{\cdot} + Iq_2^{\cdot} - (g_1 + Iq_1)\varphi_1^{\cdot} = w_2, \quad g_3^{\cdot} + I_3(q_3 + \varphi_1^{\cdot})^{\cdot} = w_3\\
&\beta_{i1}^{\cdot} + q_2\beta_{i3} - q_3\beta_{i2} = 0 \quad (i = 1, 2, 3)\\
&M_{x_1'} = -\sum_{i=1}^{3}\frac{\partial U}{\partial\beta_{i3}}\beta_{i2}, \quad M_{x_2'} = \sum_{i=1}^{3}\frac{\partial U}{\partial\beta_{i3}}\beta_{i1}, \quad M_{x_3'} = 0
\end{aligned}$$

where g_i $(i = 1, 2, 3)$ are the kinetic moments of flywheels relative to Koenig axes, w_i are control moments, and $M_{x_i'}$ are moments of gravitational forces about the same axes.

The considered steady solution is of the form

$$\begin{aligned}
&\varphi_1^{\cdot} = \omega_1 + \omega, \quad \Phi^{\cdot} = \omega_1 \qquad (1.2)\\
&q_i = 0, \quad \beta_{ij} = 0, \quad i \neq j; \quad \beta_{ii} = 1; \quad i, j = 1, 2, 3\\
&g_1 = g_2 = 0, \quad g_3 = g_3^c, \quad w_1 = w_2 = 0, \quad g_3^{\circ} = -C_3\omega_1
\end{aligned}$$

The equations of motion (1.1) admit the first integrals

$$\sum_{i=1}^{3} \beta_{ij}^2 = 1, \quad \sum_{i=1}^{3} \beta_{1i}\beta_{2i} = \sum_{i=1}^{3} \beta_{1i}\beta_{3i} = \sum_{i=1}^{3} \beta_{2i}\beta_{3i} = 0$$

and the integrals that define the constancy of projections of the gyrostat kinetic moment on the system axes $O_1X_1X_2X_3$

$$L_i + (Cq_1 + g_1)\beta_{i1} + (Cq_2 + g_2)\beta_{i2} + [C_3(q_3 + \varphi_1^{\cdot}) + g_3]\beta_{i3} = h_i \quad (1.3)$$
$$L_1 = MR^2 (\Psi^{\cdot} \sin\Phi - \Phi^{\cdot} \sin\Psi \cos\Psi \cos\Phi)$$
$$L_2 = -MR^2 (\Psi^{\cdot} \cos\Phi + \Phi^{\cdot} \sin\Psi \cos\Psi \sin\Phi)$$
$$L_3 = MR^2\Phi^{\cdot} \cos^2\Psi$$
$$h_1^{\circ} = 0, \quad h_2^{\circ} = 0, \quad h_3^{\circ} = M\sqrt{\varkappa P} + C_3(\omega_1 + \omega) + g_3^{\circ}$$

Using (1.3) for eliminating g_i from Eqs. (1.1), for the relative motion of the gyrostat we obtain equations of the form

$$(C - I)\, q_1^{\cdot} = -(C - I)\, q_2^{\cdot}\varphi_1^{\cdot} + (q_3 + \varphi_1^{\cdot}) \sum_{i=1}^{3} (h_i - L_i)\, \beta_{i2} -$$
$$q_2 \sum_{i=1}^{3} (h_i - L_i)\, \beta_{i3} + M_{x_1'} - w_1$$
$$(C - I)\, q_2^{\cdot} = (C - I)\, q_1\varphi_1^{\cdot} -$$
$$(q_3 + \varphi_1^{\cdot}) \sum_{i=1}^{3} (h_i - L_i)\, \beta_{i1} + q_1 \sum_{i=1}^{3} (h_i - L_i)\, \beta_{i3} + M_{x_2'} - w_2$$
$$(C_3 - I_3)(q_3 + \varphi_1^{\cdot})^{\cdot} = q_2 \sum_{i=1}^{3} (h_i - L_i)\, \beta_{i1} - q_1 \sum_{i=1}^{3} (h_i - L_i)\, \beta_{i2} - w_3$$

Assuming that motion (1.2) is unperturbed, we denote perturbations of variables by β_{ij}', q_i', h_i', w_i' where

$$\beta_{ij} = \beta_{ij}' \ (i \neq j), \quad \beta_{ii} = 1 + \beta_{ii}', \quad q_i = q_i'$$
$$h_i = h_i' \ (i = 1, 2), \quad h_3 = h_3^{\circ} - \sqrt{\varkappa P}\, M + h_3'$$
$$w_i = w_i' \ (i = 1, 2), \quad w_3 = -(C_3 - I_3)\varphi_1^{\cdot\cdot} + w_3'$$

Omitting the primes, we write equations of perturbed motion as

$$q_1^{\cdot} = h_{12}q_3 - (h_{13} + \omega^*)\, q_2 + \omega^* \sum_{i=1}^{3} h_{1i}\beta_{i2} + \quad (1.4)$$
$$\beta_{13}\nu \sin 2\Phi + 2\beta_{23}\nu \sin^2\Phi + v_1 + Q_1$$
$$q_2^{\cdot} = (h_{13} + \omega^*)\, q_1 - h_{11}q_3 - \omega^* \sum_{i=1}^{3} h_{1i}\beta_{i1} -$$
$$2\beta_{13}\nu \cos^2\Phi - \beta_{23}\nu \sin 2\Phi + v_2 + Q_2$$
$$q_3^{\cdot} = h_{31}q_2 - h_{32}q_1 + v_3 + Q_3$$
$$\beta_{ii}^{\cdot} = B_{ii}, \quad i = 1, 2, 3$$
$$\beta_{12}^{\cdot} = -q_3 + B_{12}, \quad \beta_{31}^{\cdot} = -q_2 + B_{31}, \quad \beta_{23}^{\cdot} = -q_1 + B_{23}$$
$$\beta_{21}^{\cdot} = q_3 + B_{21}, \quad \beta_{13}^{\cdot} = q_2 + B_{13}, \quad \beta_{32}^{\cdot} = q_1 + B_{32}$$
$$B_{i1} = q_3\beta_{i2} - q_2\beta_{i3}, \quad Q_1 = \sum_{i=1}^{3} h_{1i}B_{i1} + U_{1\beta}$$

$$Q_2 = \sum_{i=1}^{3} h_{1i} B_{i2} + U_{2\beta}, \quad Q_3 = \sum_{i=1}^{3} h_{3i} B_{i3}$$
$$h_{1j} = h_j \,/\, (C - I), \quad h_{3j} = h_j \,/\, (C_3 - I_3), \quad j = 1,\ 2$$
$$h_{13} = (h^\circ + h_3) \,/\, (C - I), \quad h_{33} = (h^\circ + h_3) \,/\, (C_3 - I_3)$$
$$\omega^* = \omega_1 + \omega, \quad \nu = {}^3/_2 \varkappa R^{-3} (C_3 - C) \,/\, (C - I)$$

where v_i are control moments related to w_i by formulas

$$(C - I) v_1 = -w_1 + \omega^* h_2$$
$$(C - I) v_2 = -w_2 - \omega^* h_1, \quad (C_3 - I_3) v_3 = -w_3$$

Note that the order of smallness of B_{ij} relative to q_i and β_{ij} is not lower than the second. The terms $U_{1\beta}$ and $U_{2\beta}$, due to gravitational moments and dependent only on β_{ij}, vanish when $\beta_{ij} = 0$ are also of the second order of smallness.

The problem of optimal stabilization is formulated as follows. We have to determine control v_i in the form of functions of variables q_i and β_{ij} so that the trivial solution of system (1.4) is asymptotically stable with respect to variables q_i, and β_{ij} and that the condition of minimum of the integral type functional

$$\int_0^\infty \Omega (q_1, q_2, q_3, \beta_{11}, \ldots, \beta_{33}, v_1, v_2, v_3, \Phi)\, d\Phi$$

2. To solve the problem of stabilization we investigate the periodic solution of the linear inhomogeneous system of the form

$$\mathbf{x}^\cdot = \mathbf{x} d + A(t)\, \mathbf{x} + \boldsymbol{\varphi}(t) \tag{2.1}$$

where $\mathbf{x}$ is a vector with components x_i ($i = 1, \ldots, n$), $d = \text{const}$ is a parameter whose magnitude will be defined later, $A(t)$ is an $n \times n$ periodic matrix of period T which satisfies the conditions of the theorem on the existence an uniqueness of solution of the differential equation, and $\boldsymbol{\varphi}(t) = \text{col}\ \{\varphi_1(t), \ldots, \varphi_n(t)\}$ is a periodic vector function of period T which has a bounded derivative.

Let us prove that the estimate

$$\| \mathbf{x}(t) + \boldsymbol{\varphi}(t) \,/\, d \| < c \,/\, d^2 \tag{2.2}$$

is valid for the periodic solution of that system.

Consider the system

$$x_i^\cdot = d x_i + \varphi_i(t), \quad i = 1, \ldots, n$$

The periodic solution of the i-the equation is of the form

$$x_i(t) = -\frac{\varphi_i(t)}{d} + d^{-1} (1 + e^{dT})^{-1} \int_0^T \varphi_i^\cdot (t + \xi)\, e^{d(T - \xi)} d\xi$$

We define the norm of $\mathbf{x}(t)$ as

$$\| \mathbf{x}(t) \| = \sum_{i=1}^{n} \max_t | x_i(t) |$$

Then

$$\| \mathbf{x}(t) + \boldsymbol{\varphi}(t) \,/\, d \| < c_1 \,/\, d^2, \quad c_1 = \| \boldsymbol{\varphi}^\cdot (t) \| \tag{2.3}$$

We use the method of successive approximations for determining the periodic solution of the system, and introduce the small parameter $\varepsilon = 1 / d$. The equation of the k-th approximation is of the form

$$\mathbf{x}^{\cdot k} = \mathbf{x}^k d + A(t)\, \mathbf{x}^{k-1} + \varphi(t), \quad \mathbf{x}^{\cdot \circ} = \mathbf{x}^\circ d + \varphi(t)$$

We shall show that the sequence $\mathbf{x}^k(t)$ converges to $\mathbf{x}(t)$. Denoting $\mathbf{x}^k(t) - \mathbf{x}^{k-1}(t) = \mathbf{y}^{k-1}(t)$, for $\mathbf{y}^k(t)$ we obtain

$$\mathbf{y}^k(t) = (1 - e^{dT})^{-1} \int_0^T A(t+\xi)\, \mathbf{y}^{k-1}(t-\xi)\, e^{d(T-\xi)} d\xi \tag{2.4}$$

$$\| \mathbf{y}^k(t) \| \leqslant \| A(t) \| \| \mathbf{y}^{k-1}(t) \| / d$$

Hence for the convergence of the sequence it is necessary that $\| A(t) \| < d$. For $\| \mathbf{y}^\circ(t) \|$ we have the estimate

$$\| \mathbf{y}^\circ(t) \| \leqslant \| A(t) \| \, \| \varphi(t) \| / d^2 \tag{2.5}$$

Using (2.4) and (2.5) we have for the remainder $\mathbf{x}(t) - \mathbf{x}^\circ(t)$ the estimate

$$\| \mathbf{x}(t) - \mathbf{x}^\circ(t) \| \leqslant \| \mathbf{x}(t) - \ldots - \mathbf{x}^k(t) + \mathbf{x}^k(t) - \ldots - \mathbf{x}^\circ(t) \| \leqslant \tag{2.6}$$

$$\sum_{k=0}^{\infty} \| \mathbf{y}^k(t) \| \leqslant \| \mathbf{y}^\circ(t) \| \sum_{k=0}^{\infty} \frac{\| A(t) \|^k}{d^k} \leqslant \frac{\| A(t) \| \| \varphi(t) \|}{d(d - \| A(t) \|)}$$

It follows from (2.6) and (2.3) that

$$\| \mathbf{x}(t) + \varphi(t) / d \| \leqslant (\| A(t) \| \| \varphi(t) \| + \| \varphi^{\cdot}(t) \|) / d^2 + O(d^3) \tag{2.7}$$

The inequality (2.2) has been thus proved. It is possible to show in a similar way the convergence of successive approximations by the substitution of variable τ of the form $d\tau / dt = \omega_1(\tau)$, where $\omega_1(\tau)$ a positive periodic function of period T_1, for the independent variable t.

3. Let us consider the linear system (1.4) without terms Q_i, which has a zero solution. In conformity with [1] we specify the integrand of the minimized functional in the form

$$\Omega_1 = F_1(q_i, \Phi) + F_2(\beta_{ij}, \Phi) + n \sum_{i=1}^{3} v_i^2 + \Lambda_1(q_i, \beta_{ij}, \Phi) \tag{3.1}$$

$$F_1 = \sum_{i,j=1}^{3} e_{ij}(\Phi)\, q_i q_j$$

$$F_2 = (4n)^{-1} \sum_{l=1}^{3} \Big(\sum_{i,j=1}^{3} a_{ij}^{(l)} \beta_{ij} \Big)^2 +$$

$$\Big[\omega^* \sum_{i=1}^{3} h_{1i}\beta_{i1} + \beta_{13}\nu(1 + \cos 2\Phi) + \nu\beta_{23} \sin 2\Phi \Big] \times$$

$$\Big(\sum_{i,j=1}^{3} a_{ij}^{(2)} \beta_{ij} \Big) - \Big[\omega^* \sum_{i=1}^{3} h_{1i}\beta_{i2} + \nu\beta_{13} \sin 2\Phi + \nu\beta_{23} \times$$

$$(1 - \cos 2\Phi) \Big] \Big(\sum_{i,j=1}^{3} a_{ij}^{(1)} \beta_{ij} \Big)$$

where F_1 and F_2 are positive definite quadratic forms with undefined coefficients $e_{ij}(\Phi)$ and $a_{ij}^{(l)}(\Phi)$ at variables q_i and β_{ij}, respectively (the positive definiteness of F_2 is proved below).

We seek an optimal Liapunov function V° of the form [1]

$$V^\circ = \sum_{i,j=1}^{3} \left[k\beta_{ij}^2 + 2\beta_{ij} \sum_{l=1}^{3} a_{ij}^{(l)}(\Phi) q_l\right] + m \sum_{i=1}^{3} q_i^2 \qquad (3.2)$$

On the basis of theorems in [3, 4] we obtain for V° the equation in partial derivatives

$$\frac{\partial V^\circ}{\partial \Phi} - \frac{1}{4n} \sum_{i=1}^{3} \left(\frac{\partial V^\circ}{\partial q_i}\right)^2 + \sum_{i=1}^{3} \left(H_i \frac{\partial V^\circ}{\partial q_i} + \lambda_i q_i\right) +$$

$$\sum_{i,j=1}^{3} \frac{\partial V^\circ}{\partial \beta_{ij}} B_{ij} + F_1(q_i, \Phi) + F_2(\beta_{ij}, \Phi) + \Lambda_1(q_i, \beta_{ij}, \Phi) = 0$$

$$H_1 = h_{12}q_3 - (h_{13} + \omega^*) q_2 + \omega^* \sum_{i=1}^{3} h_{1i}\beta_{i2} + \beta_{13}\nu \sin 2\Phi +$$
$$2\beta_{23}\nu \sin^2 \Phi$$

$$H_2 = (h_{13} + \omega^*) q_1 - h_{11}q_3 - \omega^* \sum_{i=1}^{3} h_{1i}\beta_{i1} - 2\beta_{13}\nu \cos^2 \Phi -$$
$$\beta_{23}\nu \sin 2\Phi, \quad H_3 = h_{31}q_2 - h_{32}q_1$$
$$\lambda_1 = (\partial V^\circ / \partial \beta_{32} - \partial V^\circ / \partial \beta_{23}), \quad \lambda_2 = (\partial V^\circ / \partial \beta_{13} - \partial V^\circ / \partial \beta_{31})$$
$$\lambda_3 = (\partial V^\circ / \partial \beta_{21} - \partial V^\circ / \partial \beta_{12})$$

Equating to zero the coefficients at like second order terms, we obtain systems of linear differential equations for the determination of $a_{ij}^{(l)}(\Phi)$ and an algebraic system for e_{ij}, as functions of parameters m, n, k and Φ. In particular, for $a_{13}^{(l)}$ we have the equations

$$\frac{da_{13}^{(1)}}{d\Phi} = a_{13}^{(1)}d - (h_{13} + \omega^*) a_{13}^{(2)} + h_{32}a_{13}^{(3)} - m\nu \sin 2\Phi$$

$$\frac{da_{13}^{(2)}}{d\Phi} = a_{13}^{(2)}d + (h_{13} + \omega^*) a_{13}^{(1)} - h_{31}a_{13}^{(3)} - k + 2m\nu \cos^2 \Phi$$

$$\frac{da_{13}^{(3)}}{d\Phi} = a_{13}^{(3)}d - h_{12}a_{13}^{(1)} + h_{11}a_{13}^{(2)}$$

$$A(\Phi) = \left\| \begin{matrix} 0 & -(h_{13} + \omega^*) & h_{32} \\ h_{13} + \omega^* & 0 & -h_{31} \\ -h_{13} & h_{11} & 0 \end{matrix} \right\|$$

$$\varphi(\Phi) = \mathrm{col}\,\{-m\nu \sin 2\Phi, -k + 2m\nu \cos^2 \Phi, 0\}$$

where $d = m / (2n)$.

On the basis of estimate (2.7) we obtain

$$a_{13}^{(1)} \approx m\nu \sin 2\Phi / d, \quad a_{13}^{(2)} \approx (k - 2m\nu \cos^2\Phi) / d, \quad a_{13}^{(3)} \approx 0$$

The remaining $a_{ij}^{(l)}$ are of a similar form. More exact values can be obtained by using the method of successive approximations described in Sect. 2.

Let us now prove the positive definiteness of V° and F_2. To determine the sign of F_2 with an accuracy to the first order we pass from the dependent variables β_{ij} to the independent Krylov angles in conformity with the relations

$$\beta_{13} \approx \psi, \quad \beta_{31} \approx -\psi, \quad \beta_{32} \approx \theta, \quad \beta_{23} \approx -\theta$$
$$\beta_{12} \approx \beta_{21} \approx \beta_{11} \approx \beta_{22} \approx \beta_{33} \approx 0$$

The expressions for F_2 in (3.1) and the obtained approximations of $a_{ij}^{(l)}(\Phi)$ yield

$$F_2 = n\{\theta^2 F_2^{(1)} + \psi^2 F_2^{(2)} - 4\nu\theta\psi(\omega^* h_{13} - \nu)\} \tag{3.3}$$
$$F_2^{(i)} = 4k^2 / m^2 - \omega^{*2} h_{13}^2 - 4\nu\gamma_i^2(\nu - \omega^* h_{13})$$
$$\gamma_1 = \sin\Phi, \quad \gamma_2 = \cos\Phi$$

Separating in the expression for F_2 the complete square, we find that F_2 is positive definite when

$$F_2^{(i)} > 0, \quad i = 1, 2 \tag{3.4}$$
$$4k^2 / m^2 > \omega^{*2} h_{13}^2, \quad 4k^2 / m^2 > (\omega^* h_{13} - 2\nu)^2$$

Let us show that the first two inequalities are equivalent to the two second ones. Let $\nu(\nu - \omega^* h_{13}) > 0$, then

$$4k^2 / m^2 - \omega^{*2} h_{13}^2 - 4\nu(\nu - \omega^* h_{13})\gamma_i^2 \geqslant$$
$$4k^2 / m^2 - (\omega^* h_{13} - 2\nu)^2 > 0$$

i.e. we obtain the fourth inequality. If $\nu(\nu - \omega^* h_{13}) < 0$, then

$$4k^2 / m^2 - \omega^{*2} h_{13}^2 - 4\nu(\nu - \omega^* h_{13})\gamma_i^2 \geqslant$$
$$4k^2 / m^2 - \omega^{*2} h_{13}^2 > 0$$

Finally, for the positive definiteness of F_2 we obtain

$$2k / m > \max_{\Phi \in [0, 2\pi]} |\omega^* h_{13}| \tag{3.5}$$
$$2k / m > \max_{\Phi \in [0, 2\pi]} |\omega^* h_{13} - 2\nu|$$

The coefficients in form F_1 are of the form

$$e_{11} = nd^2 + a_{23}^{(1)} - a_{32}^{(1)}, \quad e_{22} = nd^2 + a_{31}^{(2)} - a_{13}^{(2)}$$
$$e_{33} = nd^2 + a_{12}^{(3)} - a_{21}^{(3)}, \quad 2e_{12} = a_{31}^{(1)} - a_{13}^{(1)} - a_{32}^{(2)} + a_{23}^{(2)}$$
$$2e_{23} = (h_{11} - h_{31})m - a_{21}^{(2)} + a_{12}^{(2)} - a_{13}^{(3)} + a_{31}^{(3)}$$
$$2e_{13} = (h_{32} - h_{12})m - a_{21}^{(1)} + a_{12}^{(1)} - a_{32}^{(3)} + a_{23}^{(3)}$$

when d is fairly large the forms V° and F_1 are positive definite. When conditions (3.4) are satisfied the system is stable according to the first approximation. We select higher order terms Λ_1 of the form

$$\Lambda_1 = -\sum_{i,j=1}^{3} \frac{\partial V^\circ}{\partial \beta_{ij}} B_{ij}$$

On the basis of the theorem in [3] the control is of the form

$$v_i = -(2n)^{-1}\partial V^\circ / \partial q_i$$

Let us consider the complete system that defines the gyrostat relative motion. Formula (3.2) derived for function V°, obviously, solves the problem of stabilization by virtue of the complete system, if the quality criterion is of the form

$$\Omega^\circ = \Omega_1 - \sum_{i=1}^{3} Q_i \frac{\partial V^\circ}{\partial q_i}$$

Since the order of supplementary terms is not lower than the third, the fixed sign property of Ω_1 is not violated.

Thus the derived control

$$w_l = (C - I)\Big(dq_l + (2n)^{-1} \sum_{i,j=1}^{3} a_{ij}^{(l)}\beta_{ij}\Big) - (-1)^l \omega^* h_{3-l}; \quad l = 1, 2$$

$$w_3 = (C_3 - I_3)\Big(dq_3 + (2n)^{-1} \sum_{i,j=1}^{3} a_{ij}^{(3)}\beta_{ij}\Big)$$

ensures optimum stabilization of motion (1.2) when condition (3.5) is satisfied and the integrand is of the form (3.1).

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